Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2015**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

CAMBRIDGE International Examinations

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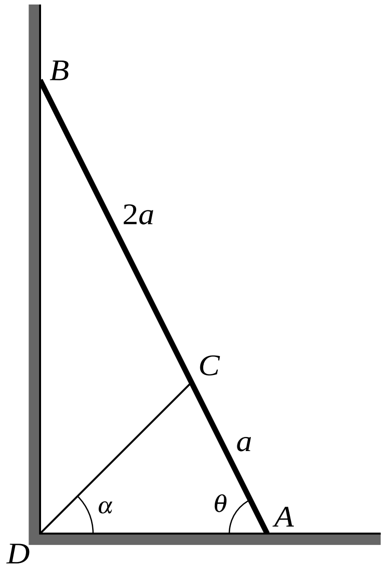


A uniform ladder AB, of length $3a$ and weight W, rests with the end A in contact with smooth horizontal ground and the end B against a smooth vertical wall. One end of a light inextensible rope is attached to the ladder at the point C, where $AC = a$. The other end of the rope is fixed to the point D at the base of the wall and the rope DC is in the same vertical plane as the ladder AB. The ladder rests in equilibrium in a vertical plane perpendicular to the wall, with the ladder making an angle θ with the horizontal and the rope making an angle α with the horizontal (see diagram). It is given that $\tan\theta = 2\tan\alpha$. Find, in terms of W and α, the tension in the rope and the magnitudes of the forces acting on the ladder at A and at B. [9]

2 A small uniform sphere A, of mass $2m$, is moving with speed u on a smooth horizontal surface when it collides directly with a small uniform sphere B, of mass m, which is at rest. The spheres have equal radii and the coefficient of restitution between them is e. Find expressions for the speeds of A and B immediately after the collision. [4]

Subsequently B collides with a vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is 0.4. After B has collided with the wall, the speeds of A and B are equal. Find e. [2]

Initially B is at a distance d from the wall. Find the distance of B from the wall when it next collides with A. [4]

3 A and B are two fixed points on a smooth horizontal surface, with $AB = 3a$ m. One end of a light elastic string, of natural length a m and modulus of elasticity mg N, is attached to the point A. The other end of this string is attached to a particle P of mass m kg. One end of a second light elastic string, of natural length ka m and modulus of elasticity $2mg$ N, is attached to B. The other end of this string is attached to P. Given that the system is in equilibrium when P is at M, the mid-point of AB, find the value of k. [3]

The particle P is released from rest at a point between A and B where both strings are taut. Show that P performs simple harmonic motion and state the period of the motion. [5]

In the case where P is released from rest at a distance $0.2a$ m from M, the speed of P is $0.7\,\mathrm{m\,s^{-1}}$ when P is $0.05a$ m from M. Find the value of a. [3]

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When P is hanging at rest vertically below O, it is projected horizontally. In the subsequent motion P completes a vertical circle. The speed of P when it is at its highest point is u. Show that the least possible value of u is $\sqrt{(ag)}$. [2]

It is now given that $u = \sqrt{(ag)}$. When P passes through the lowest point of its path, it collides with, and coalesces with, a stationary particle of mass $\frac{1}{4}m$. Find the speed of the combined particle immediately after the collision. [4]

In the subsequent motion, when OP makes an angle θ with the upward vertical the tension in the string is T. Find an expression for T in terms of m, g and θ. [5]

Find the value of $\cos\theta$ when the string becomes slack. [2]

5 A random sample of 10 observations of a normal variable X gave the following summarised data, where $\bar{x}$ is the sample mean.

$$\Sigma x = 222.8 \qquad \Sigma(x - \bar{x})^2 = 4.12$$

Find a 95% confidence interval for the population mean. [5]

6 A biased coin is tossed repeatedly until a head is obtained. The random variable X denotes the number of tosses required for a head to be obtained. The mean of X is equal to twice the variance of X. Show that the probability that a head is obtained when the coin is tossed once is $\frac{2}{3}$. [2]

Find

(i) $\mathrm{P}(X = 4)$, [1]

(ii) $\mathrm{P}(X > 4)$, [2]

(iii) the least integer N such that $\mathrm{P}(X \leqslant N) > 0.999$. [3]

7 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{21}x^2 & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^2$. Show that Y has probability density function given by

$$\mathrm{g}(y) = \begin{cases} \frac{1}{42}y^{\frac{1}{2}} & 1 \leqslant y \leqslant 16, \\ 0 & \text{otherwise.} \end{cases}$$

[5]

Find

(i) the median value of Y, [2]

(ii) the expected value of Y. [2]

8 The number of goals scored by a certain football team was recorded for each of 100 matches, and the results are summarised in the following table.

Number of goals	0	1	2	3	4	5	6 or more
Frequency	12	16	31	25	13	3	0

Fit a Poisson distribution to the data, and test its goodness of fit at the 5% significance level. [10]

9 A random sample of 8 students is chosen from those sitting examinations in both Mathematics and French. Their marks in Mathematics, x, and in French, y, are summarised as follows.

$$\Sigma x = 472 \qquad \Sigma x^2 = 29\,950 \qquad \Sigma y = 400 \qquad \Sigma y^2 = 21\,226 \qquad \Sigma xy = 24\,879$$

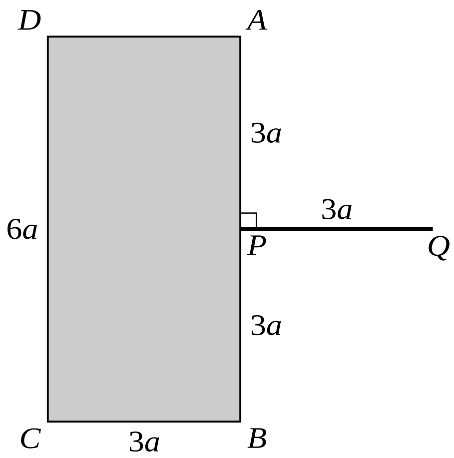
Another student scored 72 marks in the Mathematics examination but was unable to sit the French examination. Estimate the mark that this student would have obtained in the French examination. [5]

Test, at the 5% significance level, whether there is non-zero correlation between marks in Mathematics and marks in French. [6]

[Question 10 is printed on the next page.]

10 Answer only **one** of the following two alternatives.

EITHER



An object is formed by attaching a thin uniform rod PQ to a uniform rectangular lamina $ABCD$. The lamina has mass m, and $AB = DC = 6a$, $BC = AD = 3a$. The rod has mass M and length $3a$. The end P of the rod is attached to the mid-point of AB. The rod is perpendicular to AB and in the plane of the lamina (see diagram). Show that the moment of inertia of the object about a smooth horizontal axis l_1, through Q and perpendicular to the plane of the lamina, is $3(8m + M)a^2$. [4]

Show that the moment of inertia of the object about a smooth horizontal axis l_2, through the mid-point of PQ and perpendicular to the plane of the lamina, is $\frac{3}{4}(17m + M)a^2$. [2]

Find expressions for the periods of small oscillations of the object about the axes l_1 and l_2, and verify that these periods are equal when $m = M$. [8]

OR

A farmer A grows two types of potato plants, Royal and Majestic. A random sample of 10 Royal plants is taken and the potatoes from each plant are weighed. The total mass of potatoes on a plant is x kg. The data are summarised as follows.

$$\Sigma x = 42.0 \qquad \Sigma x^2 = 180.0$$

A random sample of 12 Majestic plants is taken. The total mass of potatoes on a plant is y kg. The data are summarised as follows.

$$\Sigma y = 57.6 \qquad \Sigma y^2 = 281.5$$

Test, at the 5% significance level, whether the population mean mass of potatoes from Royal plants is the same as the population mean mass of potatoes from Majestic plants. You may assume that both distributions are normal and you should state any additional assumption that you make. [9]

A neighbouring farmer B grows Crown potato plants. His plants produce 3.8 kg of potatoes per plant, on average. Farmer A claims that her Royal plants produce a higher mean mass of potatoes than Farmer B's Crown plants. Test, at the 5% significance level, whether Farmer A's claim is justified. [5]

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